Full Intertemporal Dynamic Model: Equations and Notes

Household's consumption, leisure, and saving problem:

$$maxU[(C_1, l_1) + \beta(C_2, l_2)]$$

$$s.t.: C_1 + S_1 = w_1(h - l_1) + \pi_1 - t_1$$

$$C_2 = w_2(h - l_2) + \pi_2 - t_2 + (1 + r)S_1$$

$$C_1 + \frac{C_2}{(1+r)} = w_1(h - l_1) + \frac{w_2(h - l_2)}{(1+r)} + \pi_1 + \frac{\pi_2}{(1+r)} - \left(t_1 + \frac{t_2}{(1+r)}\right)$$

$$\mathcal{L} = U(C_1, l_1) + \beta U(C_2, l_2) + \lambda\left[w_1(h - l_1) + \frac{w_2(h - l_2)}{(1+r)} + \pi_1 + \frac{\pi_2}{(1+r)} - \left(t_1 + \frac{t_2}{(1+r)}\right) - \left(C_1 + \frac{C_2}{(1+r)}\right)\right]$$

Household's first-order conditions:

$$\frac{\delta\mathcal{L}}{\delta C_1} = U_{C_1}(C_1, l_1) - \lambda = 0$$

$$\lambda = U_{C_1}(C_1, l_1)$$

$$\frac{\delta\mathcal{L}}{\delta C_2} = \beta U_{C_2}(C_2, l_2) - \frac{\lambda}{(1+r)} = 0$$

$$\lambda = \beta(1+r)U_{C_2}(C_2, l_2)$$

$$\frac{\delta\mathcal{L}}{\delta l_1} = U_{l_1}(C_1, l_1) - \lambda w_1 = 0$$

$$\lambda = \frac{U_{l_1}(C_1, l_1)}{w_1}$$

$$\frac{\delta\mathcal{L}}{\delta l_2} = \beta U_{l_2}(C_2, l_2) - \frac{\lambda w_2}{(1+r)} = 0$$

$$\lambda = \frac{\beta(1+r)U_{l_2}(C_2, l_2)}{w_2}$$

Implications of household's first-order conditions:

$$\frac{U_{l_1}}{U_{C_2}} = w_1 = MRS_{l_1 C_1}$$

$$\frac{U_{l_2}}{U_{C_2}} = w_2 = MRS_{l_2 C_2}$$

$$\frac{U_{C_1}}{\beta U_{C_2}} = (1 + r) = MRS_{C_1 C_2}$$

$$\frac{U_{l_1}}{U_{l_2}} = \frac{\beta(1 + r)w_1}{w_2} = MRS_{l_1 l_2}$$

[Note: MRS_{AB} = price of *A* in terms of *B*]

Firm's hiring and investment problem:

$$\pi_1 = A_1 F(K_1, N_1) - w_1 N_1^{(D)} - I$$

$$K_2 = (1 - \delta)K_1 + I$$

$$\pi_2 = A_2 F(K_2, N_2) - w_2 N_2^{(D)} + (1 - \delta)K_2$$

$max \Pi$ where $\Pi = \pi_1 + \frac{\pi_2}{(1+r)}$

$$\mathcal{L} = A_1 F(K_1, N_1) - w_1 N_1^{(D)} - I + \frac{A_2 F(K_2, N_2) - w_2 N_2^{(D)} + (1 - \delta)K_2}{(1 + r)} + \lambda[(1 - \delta)K_1 + I - K_2]$$

Firm's first-order conditions:

$$\frac{\delta \mathcal{L}}{\delta N_1} = A_1 F_{N_1}(K_1, N_1) - w_1 = 0$$

$$A_1 F_{N_1}(K_1, N_1) = w_1$$

$$\frac{\delta \mathcal{L}}{\delta N_2} = \frac{A_2 F_{N_2}(K_2, N_2) - w_2}{(1 + r)} = 0$$

$$A_2 F_{N_2}(K_2, N_2) = w_2$$

$$\frac{\delta \mathcal{L}}{\delta I} = -1 + \lambda = 0$$

$$\lambda = 1$$

$$\frac{\delta \mathcal{L}}{\delta K_2} = \frac{A_2 F_{K_2}(K_2, N_2) + (1-\delta)}{(1+r)} - \lambda = 0$$

Implication of firm's first-order conditions:

$\frac{\delta \mathcal{L}}{\delta K_2} = \frac{\delta \mathcal{L}}{\delta I}$ $MB_I = MC_I$

$$\frac{A_2 F_{K_2}(K_2, N_2) + (1-\delta)}{(1+r)} = 1$$

$$A_2 F_{K_2}(K_2, N_2) = (r+\delta)$$

Other definitions and budget constraints:

$$G_1 + \frac{G_2}{(1+r)} = T_1 + \frac{T_2}{(1+r)}$$

$$we = Y_1 + \frac{Y_2}{(1+r)} - \left[t_1 + \frac{t_2}{(1+r)}\right] = Y_1 + \frac{Y_2}{(1+r)} - \frac{1}{N}\left[G_1 + \frac{G_2}{(1+r)}\right]$$

Factors that shift curves:

N_S:	$-w_{t+1}$	-we	+r	
N_D:	+A	+K		
Y:	+A	+K		
Y_S:	$-w_{t+1}$	-we	+A	+K
Y_D:	I	S		

Potential shocks:

$\pm\Delta G_t$ $\pm\Delta G_{t+1}$ $\pm\Delta t_t$ $\pm\Delta t_{t+1}$ $\pm\Delta A_t$ $\pm\Delta A_{t+1}$

$\pm\Delta\delta$ $\pm\Delta\delta$